AMMY

23BMT251

UG PROGRAM (4 YEARS HONORS) WITH SINGLE MAJOR

AT THE END OF SECOND SEMESTER

MATHEMATICS - DIFFERENTIAL EQUATIONS & PROBLEM SOLVING SESSIONS (Minor)

(w.e.f. Admitted Batch 2023-24)

Time: 3 Hours **Maximum: 70 marks**

SECTION – A

Answer any FIVE Questions. **5x4=20**

1. Solve $(x^2 - 4xy - 2y^2)dx - (2x^2 + 4xy - y^2)dy = 0$
2. Solve $p^2 - 5p + 6 = 0$
3. Solve $(D^3 - 5D^2 + 8D - 4)\, y = e^{2x}$
4. Solve $(D^2 - 4D + 4)\, y = xe^{2x}$
5. Find the condition if $y = x$ *is a solution of* $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$
6. Find the integrating factor of $(xy^2 + 2x^2y^3)\, dx + (x^2y - x^3y^2)dy = 0$
7. Solve $(px - y)(py + x) = 2p$
8. Solve $(D^2 + 16)\, y = e^{-3x} + cos4x$

SECTION – B

Answer ALL Questions. **5x10=50**

9. (a) Solve $x\frac{dy}{dx} + y = y^2 logx$

(Or)

(b) Solve $x^2ydx - (x^3 + y^3)dy = 0$

10. (a) Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1$ where λ is the parameter.

(Or)

(b) Solve $y = 2xp + x^2p^4$

11. (a) Solve $(D^2 - 2D + 4)\, y = 8(x^2 + e^{2x} + Sin\, 2x)$

(Or)

(b) Solve $(D^2 - 4D + 3)\, y = Sin\, 3xCos2x$

23BMT451

UG PROGRAM (4 YEARS HONORS) WITH SINGLE MAJOR
AT THE END OF FOURTH SEMESTER
MATHEMATICS-RING THEORY & PROBLEM SOLVING SESSIONS (Minor)
(w.e.f. Admitted Batch 2023-24)

Time: 3Hours **Maximum: 70 marks**

Section –A

Answer any FIVE Questions. **5x4=20**

1. Define characteristic of a ring and give an example.
2. If A is an ideal of a ring R with unity and $1 \in A$ then show that $A = R$
3. Find all the principal ideals of $(Z_6, \oplus, \odot)$
4. Prove that every homomorphic image of a ring is a ring.
5. Prove that $f(x) = 8x^3 + 6x^2 - 9x + 24$ is irreducible over Q
6. If R is a ring and $C(R) = \{x \in R : ax = xa \, \forall a \in R\}$ then show that C(R) is a subring of R
7. Let f be a homomorphism from a ring R into a ring S. Then prove that $f(0) = 0$ and $f(-a) = -f(a), a \in R$
8. If R is a commutative ring with unity then so is $R[x]$

Section – B

Answer ALL Questions. **5x10=50**

9. a) Prove that every finite integral domain is a field.

(Or)

b) If $Q(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in Q\}$ then show that $Q(\sqrt{2})$ is a field.

10. a) Let A, B be two ideals of a ring R. Then show that $A \cup B$ is an ideal of R if and only if either $A \subseteq B$ or $B \subseteq A$.

(Or)

b) A commutative ring R with unity is a field if and only if (0) and R are the only ideals of R.

11. a) If R is a commutative ring with unity and $a \in R$ then $Ra = \{ra : r \in R\}$ is a principal ideal generated by a

(Or)

b) Prove that every field is a principal ideal ring.

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12. a) Prove that an epimorphism f from a ring R onto a ring S is an into isomorphism if and only if $Ker\, f = \{0\}$

(Or)

b) Determine the maximal ideals in the ring of integers.

13. a) State and prove Division Algorithm in the polynomial ring $F[x]$

(Or)

b) If F is a field then show that polynomial ring $F[x]$ is a principal ideal domain.

Let U_1, U_2 it is an Ideal of R.
The minimal $U_1 \subset U_2$ or $U_2 \subset U_2$
is possible
Suppose that $U_1 \not\subset U_2$ and $U_2 \not\subset U_1$
so $U_1 \not\subset U_2$ then en elu c
$b \in U_2$ and $b \notin U_2$

Given
Given condition convey

23BMT452

UG PROGRAM (4 YEARS HONORS) WITH SINGLE MAJOR

AT THE END OF FOURTH SEMESTER

MATHEMATICS - INTRODUCTION TO REAL ANALYSIS & PROBLEM SOLVING SESSIONS (Minor)

(w.e.f. Admitted Batch 2023-24)

Time: 3 Hours **Maximum: 70 marks**

SECTION – A

Answer any Five questions. **5x4=20**

1. Show that every convergent sequence is bounded.
2. Test for the convergence of $\sum(\sqrt{n^2+1}-n)$
3. Examine the continuity of the function f defined by $f(x) = |x| + |x-1|$ at $x = 0$ and 1
4. Using Lagrange's theorem show that $x > log(1+x) > \frac{x}{1+x}$ if $f(x) = log\,(1+x)\ \forall x > 0$
5. If $f(x) = 2x - 1$ on $[0,1]$ and $P\left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$, find $L(P,f)$ and $U(P,f)$
6. If $S_n = 1 + \frac{1}{2!} + \dots\dots + \frac{1}{n!}$ then show that the sequence$\{S_n\}$ is a Cauchy's sequence.
7. State and prove Leibnitz Test.
8. If $f \in R[a,b]$then show that $|f| \in R[a,b]$

SECTION – B

Answer ALL questions. **5x10=50**

9. (a) If $s_n = \left(1+\frac{1}{n}\right)^n$ then show that $\{s_n\}$ is monotonically increasing and bounded above.

(Or)

(b) Prove that a sequence is convergent if and only if it is a Cauchy sequence.

10. (a) State and prove Cauchy's n^th root test

(Or)

(b) Test for the convergence of $\sum \frac{1.3.5\ldots\ldots(2n-1)}{2.4.6\ldots\ldots 2n} x^{n-1}$ $(x > 0)$

11. (a) Prove that the function $f(x) = sin\,\frac{1}{x}(x > 0)$ is continuous on R^+ but not uniformly continuous

(Or)

(b) If f is continuous on [a,b] then show that f is bounded on [a,b]

12. (a) State and prove Rolle's theorem.

(Or)

(b) Find 'c' of Cauchy's mean value theorem for $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$ in (a, b) where $0 < a < b$

13. (a) If $f, g \in R[a, b]$ and g keeps the same sign on $[a, b]$ the show that there exists $\mu \in R$ lying between the infimum and supremum of f such that

$$\int_a^b f(x)g(x)\,dx = \mu \int_a^b g(x)\,dx$$

(Or)

(b) State and prove fundamental theorem of integral calculus

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= Subinterval

$f_1 = (0, 1/3)$ $\delta_1 - 1/3 = 1/3$

$f_2 = (1/3, 2/3)$ $\delta_2 = 2/3 - 1/3$

$f_3 = (2/3, 1)$ $\delta_3 = 1 - 2/3$

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